Marginal Analysis

Math 130 - Essentials of Calculus

23 October 2019

Revenue and Profit

Revenue, denoted by $R(q)$, is the total amount of money collected by a company after producing and selling q items, and we call R the *revenue function*.

Revenue and Profit

Revenue, denoted by $R(q)$, is the total amount of money collected by a company after producing and selling q items, and we call R the *revenue function*. Similarly with cost function, we can create the *average revenue* and *marginal revenue* functions:

Definition (Average Revenue and Marginal Revenue)

If $R(q)$ is the total revenue after producing q units of a good or service, then the average revenue per unit is

$$\frac{R(q)}{q}$$

and the marginal revenue is

$$R'(q) = \frac{dR}{dq}.$$

Revenue and Profit

Marginal revenue can be though of as approximately the additional income gained by producing and selling one additional unit (assuming the number of units produced is relatively large). If a producer always charges the same price for each unit of a product, the marginal revenue is always the same (and, in fact, is equal to the price of the object). However, with changing amounts of production, it is typical to change the price. For example, if a significantly larger number of items are produced, oversaturation can occur, driving prices down.

Revenue and Profit

Marginal revenue can be though of as approximately the additional income gained by producing and selling one additional unit (assuming the number of units produced is relatively large). If a producer always charges the same price for each unit of a product, the marginal revenue is always the same (and, in fact, is equal to the price of the object). However, with changing amounts of production, it is typical to change the price. For example, if a significantly larger number of items are produced, oversaturation can occur, driving prices down.
Regardless, an important question to ask is “how does a company maximize its profits?”

Revenue and Profit

Marginal revenue can be though of as approximately the additional income gained by producing and selling one additional unit (assuming the number of units produced is relatively large). If a producer always charges the same price for each unit of a product, the marginal revenue is always the same (and, in fact, is equal to the price of the object). However, with changing amounts of production, it is typical to change the price. For example, if a significantly larger number of items are produced, oversaturation can occur, driving prices down.
Regardless, an important question to ask is “how does a company maximize its profits?”

Definition (Profit Function)

The profit function is given by

$$P(q) = R(q) - C(q).$$

Revenue and Profit

If producing an additional unit adds more revenue than cost, it will increase profit, and therefore production should be increased.

Revenue and Profit

If producing an additional unit adds more revenue than cost, it will increase profit, and therefore production should be increased. We are basically looking for the place where the profit stops increasing. Looking at $P'(q) = R'(q) - C'(q)$, we see that $P(q)$ is increasing as long as $R'(q) > C'(q)$.

Revenue and Profit

If producing an additional unit adds more revenue than cost, it will increase profit, and therefore production should be increased. We are basically looking for the place where the profit stops increasing. Looking at $P'(q) = R'(q) - C'(q)$, we see that $P(q)$ is increasing as long as $R'(q) > C'(q)$.

To maximize profit, production should be increased to the point at which marginal revenue and marginal cost are equal.

Revenue and Profit

If producing an additional unit adds more revenue than cost, it will increase profit, and therefore production should be increased. We are basically looking for the place where the profit stops increasing. Looking at $P'(q) = R'(q) - C'(q)$, we see that $P(q)$ is increasing as long as $R'(q) > C'(q)$.

To maximize profit, production should be increased to the point at which marginal revenue and marginal cost are equal.

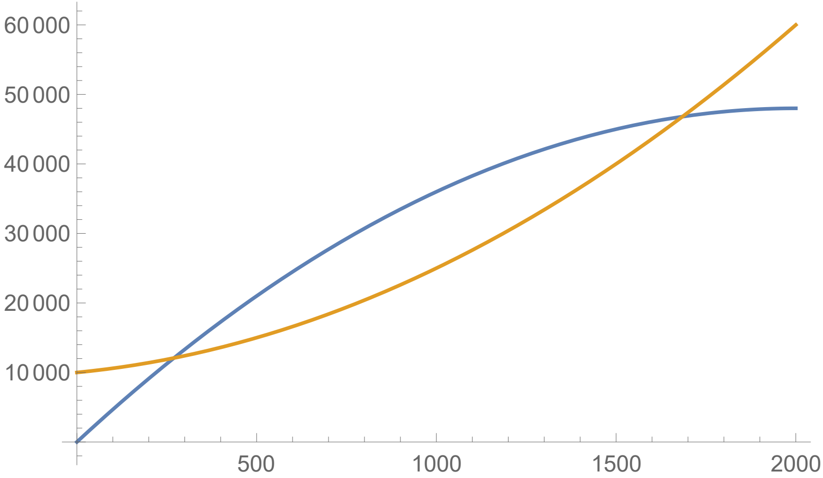
Example

In the situation of the furniture manufacturer example, suppose the company estimates that the revenue, in dollars, realized by producing q units of the chair, up to a maximum of 2000 *chairs is given by* $R(q) = 48q - 0.012q^2$. *(The cost function was given by* $C(q) = 10000 + 5q + 0.01q^2$.*)*

1. *What is the marginal revenue when* 1500 *chairs are produced?*
2. *Determine the number of chairs that the company should produce in order to maximize profit.*

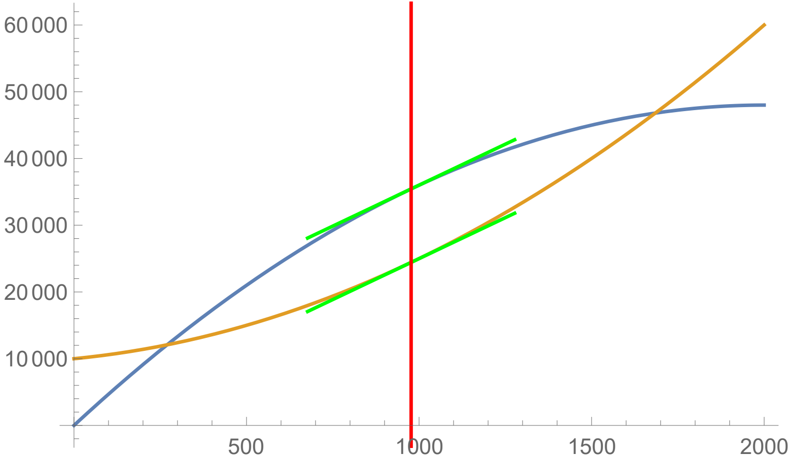
Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue).



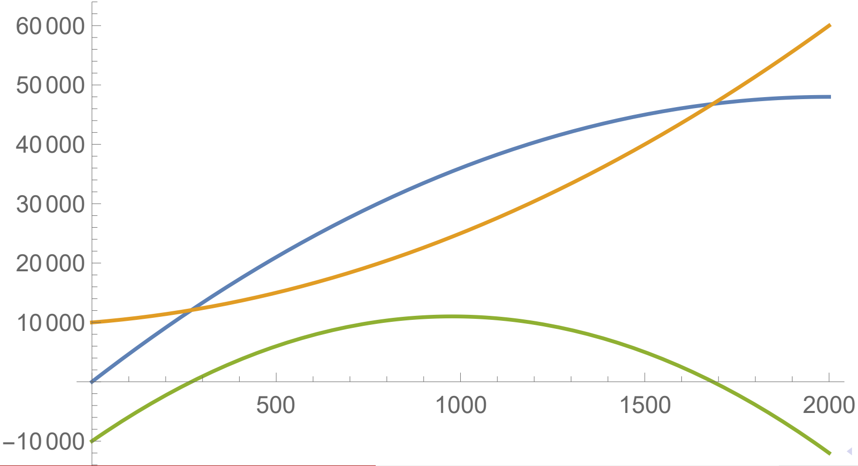
Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue), together with their tangent lines (in green) at the number we found before (in red).



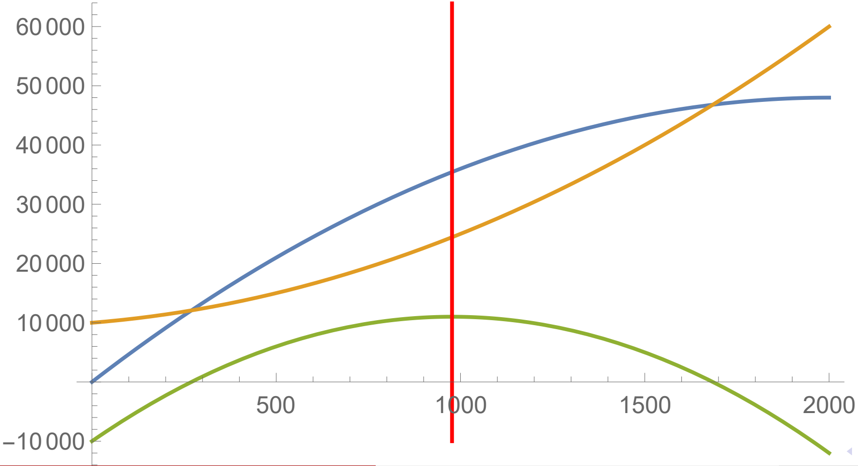
Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue), together with the profit function (in green).



Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue), together with the profit function (in green) and the red line at $q = 977$.



Now You Try It!

Example

A manufacturer of power supplies estimates that it will incur a total cost of $C(q) = 2500 + 4q + 0.005q^2$ when producing q power supplies, and it will collect $R(q) = 16q - 0.002q^2$ dollars in revenue.

1. *Write a function for the profit P the manufacturer can expect after producing q power supplies.*
2. *Find the marginal cost and marginal revenue functions.*
3. *How many power supplies should the manufacturer produce in order to maximize profit.*

Demand Curves

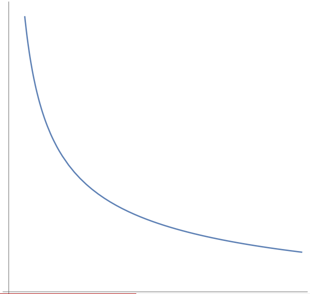
There is normally a relationship between the price of a product or service and the number of units that can be sold. Let $p = D(q)$ be the price per unit that a company can charge if it sells q units. This function D is called the **demand function** (also called a *price function*) and its graph is called the **demand curve.**

Demand Curves

There is normally a relationship between the price of a product or service and the number of units that can be sold. Let $p = D(q)$ be the price per unit that a company can charge if it sells q units. This function D is called the **demand function** (also called a *price function*) and its graph is called the **demand curve.** We expect p to be a decreasing function of q since, in order to sell more units, a lower price would be required.

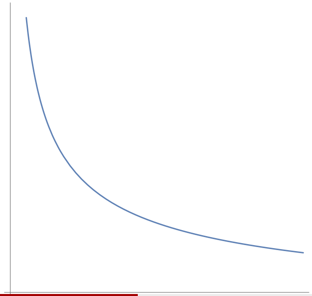
Demand Curves

There is normally a relationship between the price of a product or service and the number of units that can be sold. Let $p = D(q)$ be the price per unit that a company can charge if it sells q units. This function D is called the **demand function** (also called a *price function*) and its graph is called the **demand curve.** We expect p to be a decreasing function of q since, in order to sell more units, a lower price would be required. Here's a typical shape of a demand curve $p = D(q)$.



Demand Curves

There is normally a relationship between the price of a product or service and the number of units that can be sold. Let $p = D(q)$ be the price per unit that a company can charge if it sells q units. This function D is called the **demand function** (also called a *price function*) and its graph is called the **demand curve.** We expect p to be a decreasing function of q since, in order to sell more units, a lower price would be required. Here's a typical shape of a demand curve $p = D(q)$.



Because revenue is the number of units sold times the price per unit, the revenue can be found as

$$R(q) = q \cdot D(q).$$

Maximizing Profit

Example

A company has cost and demand functions

$$C(q) = 84 + 1.26q - 0.01q^2 + 0.00007q^3 \quad \text{and} \quad D(q) = 3.5 - 0.01q.$$

1. *If the price of each unit is* $1.20, *how many units will be sold?*

Maximizing Profit

Example

A company has cost and demand functions

$$C(q) = 84 + 1.26q - 0.01q^2 + 0.00007q^3 \quad \text{and} \quad D(q) = 3.5 - 0.01q.$$

1. *If the price of each unit is $1.20, how many units will be sold?*
2. *Determine the production level that will maximize profit for the company.*

Now You Try It!

Example

A company has cost and demand functions

$$C(q) = 680 + 4q + 0.01q^2 \quad \text{and} \quad p = 12 - \frac{q}{500}.$$

Find the production level that will maximize profit.

A More Complicated Relationship

Example

A company estimates that the number of units x of a new product, measured in thousands, that can be sold and the price p, in dollars, of each unit are related by the equation

$$px^2 + 15px = 30000.$$

Find $\dfrac{dx}{dp}$ when the price is \$30 per unit.